M463 Homework 9

Enrique Areyan
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Suppose a fair die is rolled 300 times. For each i, $1 \leq i \leq 300$, let X_i be the indicator of a six coming up on the ith roll, that is:

$$X_i = \begin{cases} 1 \text{ if the result of the } i\text{th roll was a six} \\ 0 \text{ otherwise} \end{cases}$$

Use the linearity of expected value to evaluate $E(X_1 + X_2 + X_3 + \cdots + X_{300})$

Solution:

First note that the distribution of X_i is $P(X_i = 1) = \dfrac{1}{6}$ and $P(X_i = 0) = \dfrac{5}{6}$ for any i, $1 \leq i \leq 300$. In other words, X_i is a Bernoulli trial with probability of success $p = \dfrac{1}{6}$. Therefore:

$$E(X_i) = \sum_{all\ x} x \cdot P(X = x) = \sum_{0}^{1} x \cdot P(X = x) = 0 \cdot \frac{5}{6} + 1 \cdot \frac{1}{6} = \frac{1}{6}$$

Using the linearity of expected value we can evaluate the following:

$$\begin{aligned} E(X_1 + X_2 + X_3 + \cdots + X_{300}) &= E(X_1) + E(X_2) + E(X_3) + \cdots + E(X_{300}) \\ &= \frac{1}{6} + \frac{1}{6} + \cdots + \frac{1}{6} \\ &= \frac{300}{6} \\ &= \boxed{50} \end{aligned}$$

This is consistent with the fact that these 300 trials can be modeled with a Binomial distribution as follow: Let $Y = X_1 + X_2 + \cdots + X_{300}$, where $Y \sim Binomial(n = 300, p = 1/6)$. Hence, $E(Y) = np = \frac{300}{6} = \boxed{50}$